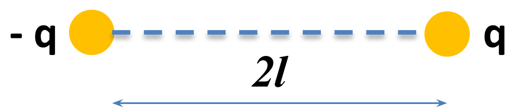
Electric Dipole

Electric dipole

A system of two equal and opposite charges separated by a small distance is called electric dipole, shown in figure.



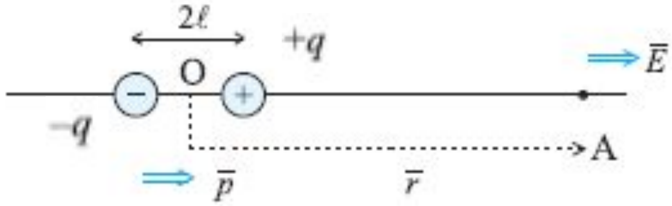
Every dipole has a characteristic property called dipole moment. It is defined as the product of magnitude of either charge and the separation between the charges, given as $\vec{p} = q(2l)$ directed from negative to positive charge,

Dimension : [LTA],

Units : coulomb × metre (or C-m)

A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C located at points A(0, 0, 0.15 m)and B(0, 0, + 0.15 m) respectively. What is the total charge and electric dipole moment of the system? [Ans. 0, 7.5×10^{-8} Cm]

Electric field at axis of electric dipole



Electric field of a short dipole on its axis at a point A at a distance r from dipole (l << r) :

$$E_A = \frac{q}{4\pi\varepsilon_0 (r-\ell)^2} - \frac{q}{4\pi\varepsilon_0 (r+\ell)^2}$$

$$E_A = \frac{4q\ell r}{4\pi\varepsilon_0 \left(r^2-\ell^2\right)^2}$$

$$\overline{E}_A = \frac{2\overline{p}}{4\pi\varepsilon_0 r^3}$$

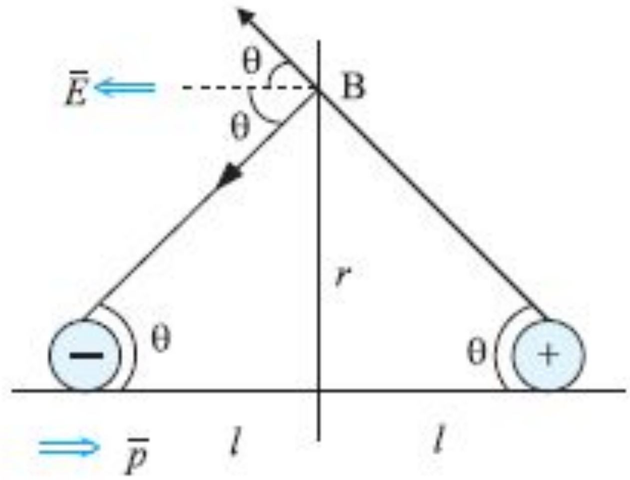
Electric field at equator :

Electric field at a point distance r from the centre of the short dipole (l << r)

$$E_B = 2\frac{q}{4\pi\varepsilon_0\left(r^2+\ell^2\right)}\cos\theta$$

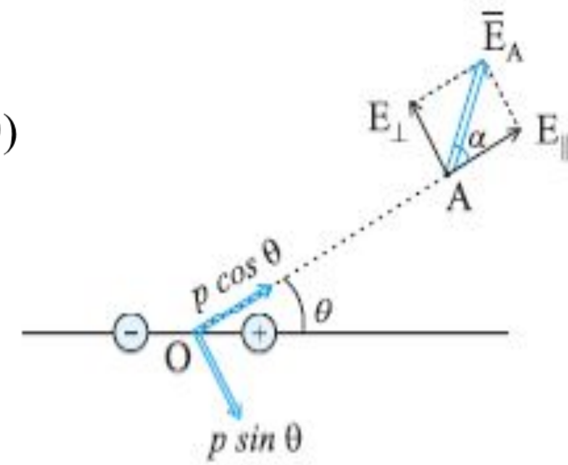
$$E_B = \frac{2q}{4\pi\varepsilon_0\left(r^2+\ell^2\right)}\frac{\ell}{\sqrt{\ell^2+r^2}}$$

$$\overline{E_B} = \frac{-\overline{p}}{4\pi\varepsilon_0 r^3}$$



Electric field at any point A (r, θ) due to short dipole :

Let A be a point at a distance r from the mid-point O of the short dipole. Let θ be the angle between OA and the dipole moment p.
In order to find the field at A, we resolve the dipole moment p into its components Along OA and perpendicular to OA. These two components are pcosθ and psinθ respectively. The point A now lies on the axis of the dipole pcosθ and on the right bisector of the dipole psinθ .



$E_\parallel$ = field component along OA (created by pcosθ)

$$\vec{E}_\parallel = \frac{2p\cos\theta}{4\pi\varepsilon_\circ r^3}\ along\ \overrightarrow{OA}$$

$\vec{E}_\perp$ = field component ⊥ to OA created by psinθ

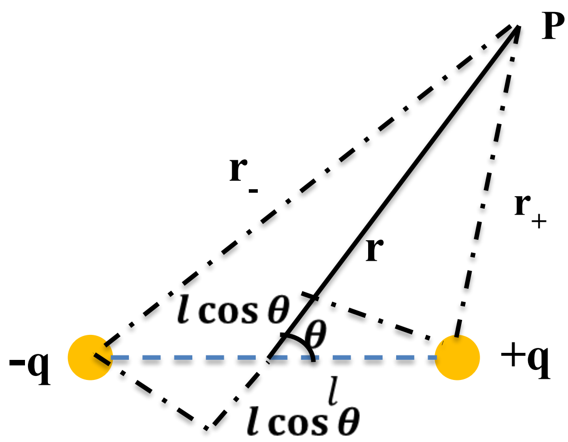
$$\vec{E}_\perp = \frac{p\sin\theta}{4\pi\varepsilon_\circ r^3}\ \perp\ to\ \overrightarrow{OA}$$

$$\vec{E}_A = \sqrt{E_\parallel^{\,2} + E_\perp^{\,2}} = \frac{p}{4\pi\varepsilon_\circ r^3}\sqrt{1+3cos^2\theta}$$

$$\tan\alpha = \frac{E_\perp}{E_\parallel} = \frac{1}{2}\tan\theta$$

$$\text{Or, } \alpha = \tan^{-1}\left(\frac{\tan\theta}{2}\right)$$

Electric potential at any point A (r, θ) due to short dipole



Electric potential at P = V = $V_+ + V_-$

$$= \frac{kq}{r_+} - \frac{kq}{r_-}$$

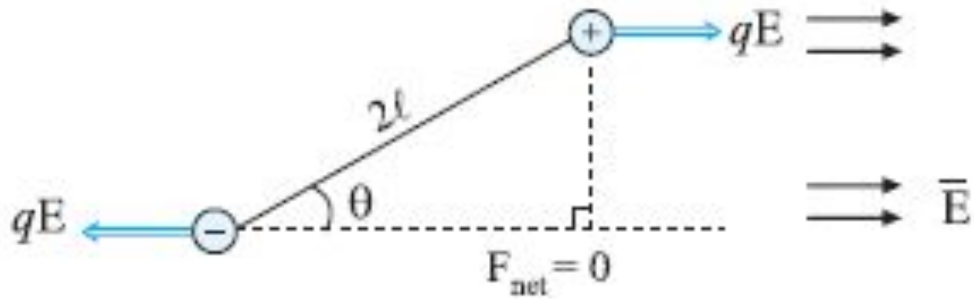
$$= \frac{kq}{r - l\cos\theta} - \frac{kq}{r + l\cos\theta} = \frac{k2lq\cos\theta}{r^2 - l^2\cos^2\theta} = \frac{kp\cos\theta}{r^2} \qquad \text{since}, l \ll r$$

If $\theta = 0°$ then $V = \frac{kp}{r^2}$ and if $\theta = 90°$ then $V = 0$

Dipole in an external uniform electric field

If a dipole is placed in a uniform electric field E

1. force on the dipole is zero.
2. torque on the dipole is given as $\tau = qE2l\sin\theta$



$$\tau = magnitude\ of\ the\ force \times perpendicular\ distance\ between\ the\ line\ of\ forces$$

$$= qE2l\sin\theta$$

$$= (q2l)E\sin\theta$$

$$= pE\sin\theta \qquad \vec{\boldsymbol{\tau}} = \vec{\boldsymbol{p}} \times \vec{\boldsymbol{E}}$$

Work done in Rotation of a Dipole in Electric field

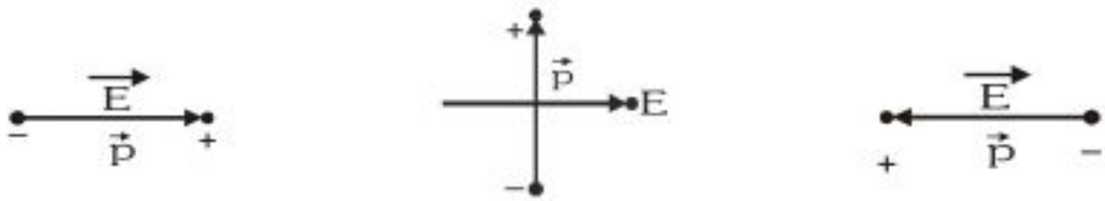
When a dipole is placed in an electric field at an angle θ, the torque on it due to Electric field is $\tau = pE\sin\theta$

Work done in rotating an electric dipole from θ_1 to θ_2[uniform field]

$$dW = \tau d\theta$$

$$\text{So,}\quad W = \int_{\theta_1}^{\theta_2} \tau d\theta = \int_{\theta_1}^{\theta_2} pESin\theta d\theta = pE(\cos\theta_1 - \cos\theta_2)$$

If a dipole is rotated from field direction ($\theta = 0°$) to θ then W= $pE(1 - cos\theta)$



$\theta = 0$	$\theta = 90$	$\theta = 180$
τ = minimum = 0	τ = maximum = pE	τ = minimum = 0
W = minimum = 0	W = pE	W = maximum = 2pE

Electrostatic potential energy

Electrostatic potential energy of a dipole placed in a uniform field is defined as work done in rotating a dipole from a direction perpendicular to the field to the given direction i.e.

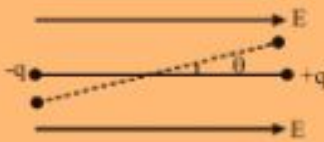
$$W = \int_{90^\circ}^{\theta} \tau d\theta = \int_{90^\circ}^{\theta} pESin\theta d\theta = pE(\cos 90^\circ - \cos\theta) = -pEcos\theta = -\vec{p}.\vec{E}$$

A short electric dipole is situated at the origin of coordinate axis with its axis along x–axis and equator along y–axis. It is found that the magnitudes of the electric intensity and electric potential due to the dipole are equal at a point distant r = 5 m from origin. Find the position vector of the point in first quadrant.

$$\because |E_P| = |V_P| \therefore \frac{kp}{r^3}\sqrt{1+3\cos^2\theta} = \frac{kp\cos\theta}{r^2} \Rightarrow 1+3\cos^2\theta = 5\cos^2\theta \Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^0$$

Position vector $\vec{r}$ of point P is $\vec{r} = \frac{\sqrt{5}}{2}\left(\hat{i}+\hat{j}\right)$

Figure shows an electric dipole formed by two particles fixed at the ends of a light rod of length l. The mass of each particle is m and the chargers are $-q$ and $+q$. The system is placed in such a way that the dipole axis is parallel to a uniform electric field E that exists in the region. The dipole is slightly rotated about its centre and released. Show that for small angular displacement, the motion is angular simple harmonic and find its time period.



Suppose, the dipole axis makes an angle θ with the electric field at an instant. The magnitude of the torque on it is

$$|\vec{\tau}| = |\vec{P} \times \vec{E}|$$

$$= ql\,E\sin\theta$$

This torque will be restoring & tend to rotate the elipole back towards the electric field. Also, for small angular displacement $\sin\theta = \theta$ so that

$$\tau = -qlE\theta$$

It the moment of inertia of the body about OA is I, the angular acceleration becomes.

$$\alpha = \frac{\tau}{I} = -\frac{qlE}{I}\theta \qquad\qquad \alpha = -\omega^2\theta$$

where $\omega^2 = \dfrac{qlE}{I}$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{qlE}}$$

Now, moment of inertia of the system about the axis of rotation is

$$I = 2m\left(\frac{l}{2}\right)^2 - \frac{ml^2}{2}$$

So, $\quad T = 2\pi\sqrt{\dfrac{ml}{2qE}}$.